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On Nonlinear Field Theories of Elementary Particles[1,2]

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Abstract

A nonlinear field equation has been derived from a gauge formalism. It has been shown that a soluble nonlinear equation, the sine–Gordon equation, can be obtained from the general nonlinear equation. The physical interpretation is given.

A possibility of describing elementary particles in terms of a fundamental field in the nonlinear spinor field formalism has been proposed by Heisenberg some time ago. The essential feature of this theory is that the masses of particles can be calculated (contrary to the conventional field theory where the mass is represented by a parameter) by means of the self-interactions of fundamental fields. In this theory, the divergence difficulties can be remedied too by the presence of the universal length which was originally introduced in the self-interaction term where it plays the role of scaling the generated masses (Heisenberg, 1966; Kim, 1973). In this program, then, every elementary particle can, in principle, be represented as excited states (in terms of the mass spectrum) of a fundamental field. One of the characteristics of the formalism is that the fundamental field is chosen as a fermion because of its elementarity. This last conjecture was challenged by Kalnay (1974) on the basis that the fermion can be described in terms of the boson field and c-number. Thus, the simplest universal field equation might not necessarily be the spinor field equation, but might very well be the scalar field equation of the type

$$\Box\phi \sim l_0^2(\phi^*\phi)\phi \tag{1}$$

where l_0 is universal length.

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This view is strengthened by recent works on supergauge formalism (Wess and Zumino, 1974) in which the possible transformation from fermion to boson field (and vice versa) has been discussed. In a study of the sine-Gordon equation, Coleman (1975) has identified the quantum soliton, a solution of the sine-Gordon equation, as a massive Thirring field by conjecturing the soliton as a fermion (rather than as a boson). This nonlinear scalar field model has also one more advantage, as we will show next, that of leading to a soluble equation.

For these reasons, Heisenberg's original motivation for choosing the spinor field equation as the only equation describing the fundamental field should be open to questions again.

The nonlinear Klein-Gordon equation (1) offers, in any event, an interesting alternative point of view to the unified field model of elementary particles. This equation contains such beautiful features as the divergence-free and mass-generating mechanism. In this note, we will show that the more rigorous Klein-Gordon type nonlinear field equation (with the desirable features, of course) can be derived in terms of a renormalizable and gauge invariant formalism.

The relevant Lagrangian is

$$L = -\tfrac{1}{2}(\partial_\mu \phi)^2 + \mu^2 |\phi|^2 - g |\phi|^4 \qquad (2)$$

The ϕ^4 term represents a self-interaction and plays, at the same time, the role of canceling the divergence arising from the ϕ^2 term. The Lagrangian is invariant under gauge transformation $\phi \to e^{i\alpha x}\phi'$.

The Lagrangian (2) leads to a nonlinear equation

$$\Box\phi = \mu^2\phi - g(\phi^*\phi)\phi \qquad (3)$$

This equation describes the massive scalar particles where the nonlinear term represents the self-interaction of the particles under consideration. The self-interaction, together with the linear interaction, may give us a clue for interpreting the mass spectrum of elementary particles. We may classify the type of interaction in equation (3) as follows:

(a) The magnitude of the linear interaction is bigger than the nonlinear interaction. The ordinary Klein-Gordon equation is a special case of this type of interaction in which the nonlinear interaction vanishes.

(b) The magnitude of the linear interaction is equal to that of the nonlinear interaction. In this case, the equation describes massless particles.

(c) The magnitude of the linear interaction is smaller than that of the nonlinear interaction. In this limit, equation (3) can be reduced to equation (1).

The resulting sum, difference of magnitude of interaction (between linear and nonlinear interaction) in equation (3) may be interpreted as the mass (rather, proportional to the mass) of the particle under consideration.

We will show now how the presence of the linear term in the present equation (3) [different from equation (1)] enables us to arrive at a soluble equation.

By rescaling of the parameter of equation (3)

$$\Box\phi = \mu^2 [\phi - (g/\mu^2)\phi^3] \tag{4}$$

we get the general sine-Gordon equation

$$\Box\phi \simeq \mu^2 \sin\phi \tag{5}$$

where we have used the condition

$$g/\mu^2 = 1/3!$$

The solution of the sine-Gordon equation is well studied. The analytic solution in one space and one time dimension, in particular, is well known (Scott *et al.*, 1973). Recently, Coleman derived the sine-Gordon equation in a somewhat different way. The Lagrangian he used is

$$L = \tfrac{1}{2}(\partial_\mu\phi)^2 + m^2 \cos\beta\phi + \gamma_0 \tag{6}$$

where $m^2 \cos\beta\phi$ is an interaction density and m, β, and γ_0 are real parameters. The Lagrangian (6) leads to the sine-Gordon equation

$$\Box\phi = m^2 \sin\beta\phi \tag{7}$$

By comparing equation (6) with the simplest soluble nonlinear spinor equation which describes a massive fermion field (Thirring, 1958), Coleman was able to identify the solution of the sine-Gordon equation (7) as a massive fermion Thirring field. The Thirring Lagrangian in question is

$$L = i\bar{\psi}\gamma_\mu\delta^\mu\psi - \tfrac{1}{2}gj^\mu j_\mu - m\sigma \tag{8}$$

where $j_\mu = \bar{\psi}\gamma_\mu\psi$ is current and $m\sigma$ is a renormalization term which is put in by hand.

It seems that the present nonlinear field equation (3) is more rigorous and more informative than the simple nonlinear scalar field equation (1). (This comment is, in principle, also valid when applied to the nonlinear spinor field equation with and without linear term.)

The equation (3) has all the features of equation (1). It provides a divergence-free theory and the ability to explain the mass spectrum of elementary particles within this formalism. In addition to this, it can be derived naturally from a gauge formalism in which the renormalizability and gauge invariance are guaranteed. Furthermore, it can provide an interesting interpretation of the particle creation and annihilation, thanks to its linear term. The difference between the magnitude of the self interaction and that of the linear interaction is proportional to the mass (or masses) of the particle under consideration, thus making it possible to define the particle as massless (difference is zero), a particle (difference is positive), or an antiparticle (difference is negative). Many particle creations can also be explained by assuming that the universal length scales the interaction in such a way that it is an integer number of particles.

In concluding we may remark on the relationship between the two nonlinear field theories. The nonlinear spinor and the nonlinear scalar field equation are compatible and do not exclude each other. Neither one is more fundamental than the other. In the nonlinear scalar field model, the boson is a fundamental field and the fermion is a composite particle state (of boson), while in the nonlinear spinor field equation, the fermion is fundamental and the boson is a fermion and antifermion bound state.

It seems that the nonlinear spinor field equation and the nonlinear scalar field equation both describe a single particle of different states. These conclusions lead us to believe that the boson and fermion are probably a single particle of different states.

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